**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. **0.2676**
4. 0.5
5. 0.6987
6. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
7. More employees at the processing center are older than 44 than between 38 and 44.
8. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Ans - Mean = 38**

**SD = 6**

**Z score = (Value - Mean)/SD**

**Z score for 44 = (44 - 38)/6 = 1 => 84.13 %**

**People above 44 age = 100 - 84.13 = 15.87% ≈ 63 out of 400**

**Z score for 38 = (38 - 38)/6 = 0 => 50%**

**Hence People between 38 & 44 age = 84.13 - 50 = 34.13 % ≈ 137 out of 400**

* **Hence More employees at the processing center are older than 44 than between 38 and 44. is FALSE**

**Z score for 30 = (30 - 38)/6 = -1.33 = 9.15 % ≈ 36 out of 400**

* **Hence A training program for employees under the age of 30 at the center would be expected to attract about 36 employees - TRUE**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans - **As we know that if X ∼ N(µ1, σ1^2 ), and Y ∼ N(µ2, σ2^2 ) are two independent random variables then X + Y ∼ N(µ1 + µ2, σ1^2 + σ2^2 ) , and X − Y ∼ N(µ1 − µ2, σ1^2 + σ2^2 )**

**Similarly if Z = aX + bY , where X and Y are as defined above, i.e Z is linear combination of X and Y , then Z ∼ N(aµ1 + bµ2, a^2σ1^2 + b^2σ2^2 ).**

**Therefore in the question**

**2X1~ N(2 u,4 σ^2) and**

**X1+X2 ~ N(µ + µ, σ^2 + σ^2 ) ~ N(2 u, 2σ^2 )**

**2X1-(X1+X2) = N( 4µ,6 σ^2)**

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans- A=PROBABILITY toward the left=-0.01/2=0.005

B=probability toward the right =0.01/2=0.005

Z=(x- μ)/ σ

X=z\* σ+ μ

X=(-0.005)\*20+200=-(-2.57)\*20+100=151.4

Y==(-0.005)\*20+200=(-2.57)\*20+100=48.6

* 90.5, 105.9
* 80.2, 119.8
* 22, 78

**48.5, 151.5**

* 90.1, 109.9

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

ANS -: **(90,990)**

1. Specify the 5th percentile of profit (in Rupees) for the company

ANS:- 202.5 millon Rupees

1. Which of the two divisions has a larger probability of making a loss in a given year?

ANS:-

For division1= Z score for a profit of zero: Z=(X-µ)/ *=>*  (0-5)/3 => -1.66=0.0485

> stats.norm.cdf(0, loc =5, scale = 3 )

0.04779035

For division2= Z score for a profit of zero: Z=(X-µ)/ =(0-7)/4 => -1.75= .0401

> stats.norm.cdf(0, loc =7, scale = 4 )

0.04005916

Division2 has a higher probability of making a loss.